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A Systematic Review of Fractal Theory and its Application in Geography and Urban Planning

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Research Article

Abstract

Fractal has been considered as an optimized structure of nature and an effective tool for depicting spatial complexity. A city can be deemed as a fractal system with self-similarity, and are, therefore, ideal candidates for fractal analysis. Recent decades have witnessed great strides in research on fractal cities in various aspects of urban forms and structures as well as urban development dynamics. Considering the deep and wide-ranging applications of fractal theory in explaining local and spatial complexities, this study aimed at conducting a systematic review and content analysis of research conducted on fractal in order to pinpoint practical and theoretical frameworks they can provide to specialists in the fields of geography and urban planning. To this end, a systematic review was carried out in three general stages of study selection, evaluation, and synthesis of findings. Results showed that developing and emerging concepts in theoretical and experimental studies, which has sought to explain the roles and applications of fractal theory in research and simulations of the built environment could be classified in 10 areas of psychology, architecture, urban design, urban form and structure, urban boundary shape, land use, urban traffic, urban growth modeling, urban hierarchy, and benefits of fractals. Studies related to urban form and structure (N= 37) accounted for the largest share of studies. In addition, key methods leading to the development of fractal studies in the existing literature were use of fractal dimension and Hausdorff dimension (34%), boundary dimension (24%), and network or box-counting dimension (14%) followed by power law (PL) distributions, power laws, and power-scale law (7%).

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1. Introduction

Euclidean geometry has had a decisive role in shaping our perceptions of nature and space for more than two thousand years. It has been influential to the extent that it has turned into an ideology, which imposes a logical pattern on nature, instead of proving a realistic description thereof. The revolutionary nature of fractal geometry lies in providing a more accurate description of nature. In fact, Mandelbrot coined the word fractal from the Latin word Fractus, meaning broken, irregular, and fractured. Mathematicians using fractal geometry focus on describing nature instead of governing it (Blanco Rivero, 2019). Examining the world around us, one can see a certain regularity in the complex system of nature and the human mind, which is necessary for the establishment of order and balance in life. Therefore, decision-making in different situations requires the mental simplification of complexities to find a kind of regularity in order to identify and remember patterns, which in turn, can lead to conscious discovery of design patterns (Sharif and Mohammad Ali Nejad, 2: 1). The fractal dimension is a measure of the extent to which a self-similar figure is "complicated" (El-Darwish, 2019).

Fractal theory emerged after the theories of "complexity" and "chaos" in the 1970s (Kiani and Amiriparyan, 2016). The shape of the fractal geometry originates from growing and developing or dividing recursive geometric series (Vaughan and Ostwald, 2016). Today, fractal geometry is a powerful tool that has long been used by researchers in the field of urban studies (Arlinghaus, 1985; Arlinghaus and Arlinghaus, 1989; Batty, 1995; Batty and Longley, 1994; Benguigui and Daoud, 1991; Thomas et al., 2010). Fractals have fractional dimensions with self-similar properties (Falconer, 1997).

Fractal is an optimized structure of nature and an effective tool for depicting spatial complexity (Liu and Chen, 2007). Fractal geometry is a powerful tool in spatial analysis and can be seen as a new look at urban and regional systems (Batty, 2008). It is argued that a city can be considered as a fractal system with self-similar properties (Batty, 2008; Batty, 2005; Benguigui et al., 2000; Chen, 2010; Feng and Chen, 2010; Thomas et al., 2007; 2008). Research on fractal cities can improve the effective use of urban geographic space (Chen, 2005). Analysis of urban forms and structures can reveal important geographical information about urban spatial development and regional urbanization. Fractal dimensions of urban form as stated in fractal theory have attracted much attention leading to many related studies. Fractal dimensions are effective measures for fractals because they can compact large amounts of spatial data into simple numbers to uncover the hidden spatial information behind the city (Chen and Huang, 2019).

Fractal city studies can lead to a new urban theory. Urban geography studies should explore simple and complex aspects and the relationship between complexity and simplicity of places (Chen, 2017). Given the deep and wide-ranging applications of fractal theory in explaining local and spatial complexities, this study aimed to answer the following question: "What practical and theoretical frameworks useful for specialists in the fields of geography and urban planning can be achieved via a systematic review and content analysis of research conducted on fractals?"

2. Literature Review

2.1. Systematic Review Steps

This section comprises a systematic review of previous studies on the research topic. This method allows for summarizing, interpreting and understanding the patterns in previous research findings as well as the possibility for objective representation of data (Lipsey and Wilson, 2001). Thus, steps of a systematic review are:

- Specification of the research topic;
- Statement of a research topic or question and explanation of the importance of using a systematic review;
- Detailed statement of study selection criteria; and
- Systematic search for related studies

which together constitute the "collection" stage.

The above steps are followed by:

- Assessment of the eligibility of studies (to exclude low quality studies and publications and those that are biased);
- Preparation of a summary of findings; and
- Systematic extraction of data through a content analysis approach;

which together constitute the "appraisal" stage.

Then, the review continues with:

- Descriptive analysis of studies (classification of studies based on different criteria via descriptive investigation);
- Narrative analysis and interpretation of studies; and
- Graphical and tabular presentation.

which together form the "synthesis" stage (Figure 1).

Fractal research in urban planning and design has a long history. In this part of research, a systematic review of studies in this area was performed. The word Fractal was searched in the Science Direct database resulting in more than 87,000 articles as of October 10, 2020. Other keywords and phrases searched were: "fractal", "fractals", "fractal in city", "fractal in planning" and "fractal in urban design". After limiting the searched terms, the number of studies reduced significantly and the focus shifted to geography and urban planning. It should be noted that other databases used in this part of research were: Web of Science, Science Direct, and Scopus. In addition, Iranian databases of SID, Magiran, and Noormags were searched. As expected, a large proportion of these studies were those in other disciplines and sciences

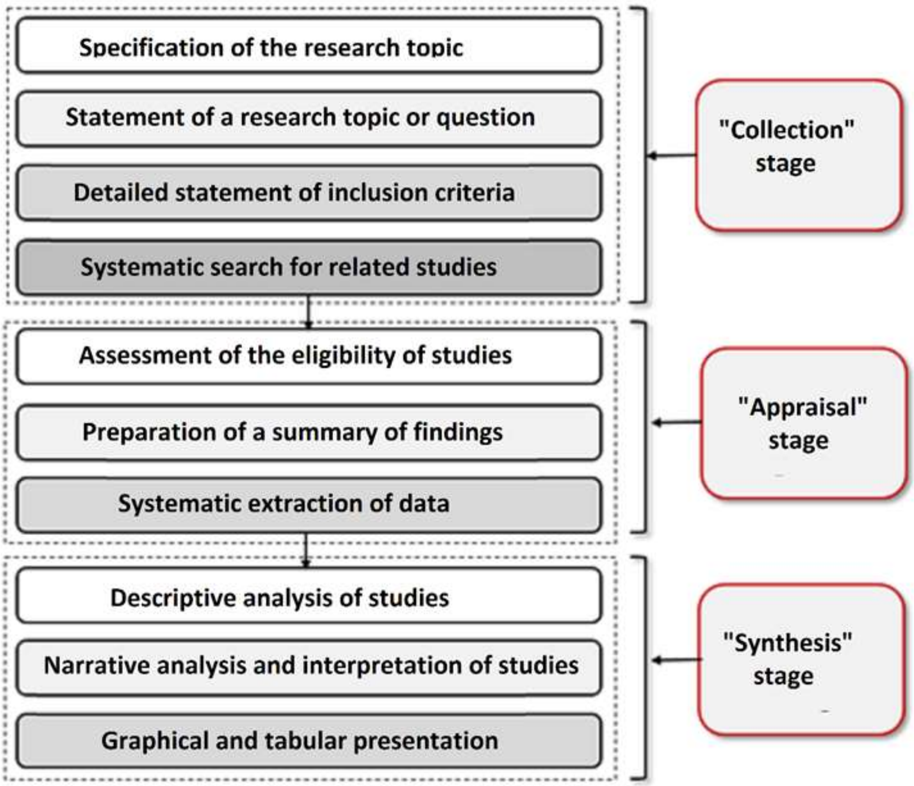

Fig 1 Stages of systematic review of fractal research in urban planning (Source: Authors)

2.2. Systematic Review Questions

Questions had been formulated before the systematic review strategy and study selection criteria were stated. Two research questions were asked during the review:

1) What are the emerging concepts in theoretical and experimental studies that explain the role of application of fractal theory in the study and simulation of built environments?

2) What key theories and methods have led to the development of fractal studies in the literature on architecture and urban design and planning?

In the following sections, an attempt has been made to give comprehensive and appropriate answers to the mentioned questions.

2.3. Systematic Review Strategy and Study Selection Criteria

As studies can be selected from published and unpublished journals, reports, books, dissertations, technical reports, conference presentations, etc., the researcher did not have limitations in selecting studies in the first step. Therefore, inclusion criteria were theoretical and practical studies related to the searched keywords, and exclusion criteria included publications that did not have a valid and academic database, such as news, periodicals, non-English language publications, articles in which bias was observed, and articles that did not use up-to-date data. In addition, studies that were outside the scope of the research and articles that were not available in full text were also excluded from the review at this stage (Lipsey and Wilson, 2001).

In the first step, all studies related to the subject were searched as explained in the mentioned databases (1079 studies). In the next step, the study titles were reviewed and articles that were not related to the fields of architecture, urban geography, and urban planning, design, and management were excluded from the sample (352 studies). Afterward, studies whose full texts were not available were excluded leading to a drop in studies (179 studies). After that, abstracts of the remaining studies were studied and those that were not valid were excluded (143 studies). The final selection was done by studying the full texts of selected studies (102 studies, including 91 external studies and 11 domestic studies). In the following sections, the most important studies are discussed with emphasis on their methodology, criteria and indices, as well as their findings and results (Figure 2).

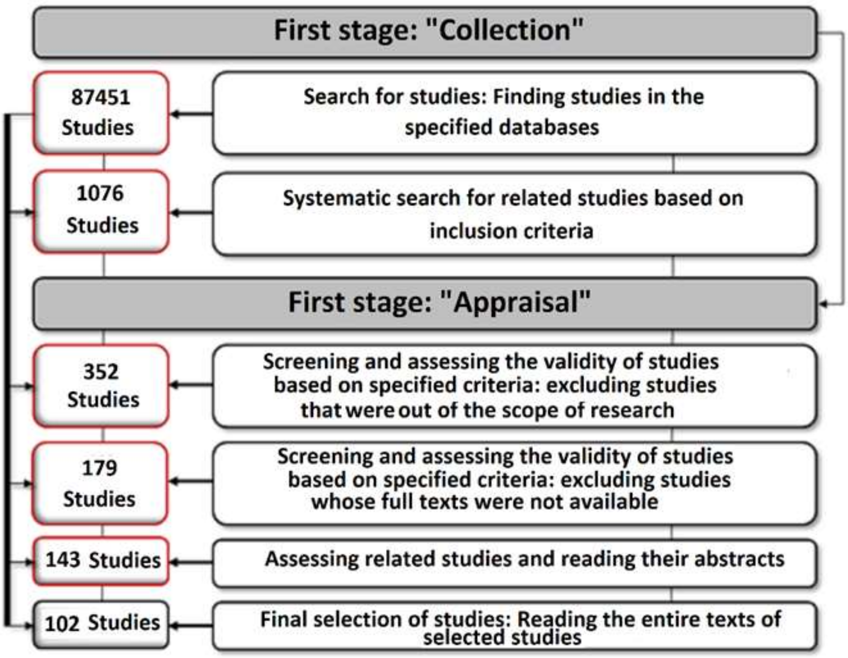

Fig 2 Systematic Review Strategy and Study Selection Criteria (Source: Authors)

In a study entitled "Multi-fractal measures of city-size distributions based on the three-parameter Zipf model", Chen and Zhou (2004) investigated the relationship between urban hierarchies and self-organized multi-fractal networks. To this end, they uncovered the multi-fractal nature of the urban hierarchy, which is related to the rank-size distribution of cities in an area. Putting the questions of "how can we interpret the multi-fractal structure of urban hierarchies, or where is the underlying rationale of the multi-fractal measures of urban systems?" (p. 793), they tried to clarify the underlying logical relationships (Chen and Zhou, 2004). In "Connecting the Fractal City", Salingaros (2004) describes distinct types of cities through connective geometry, which includes different degrees of urban life, and defines fractal, scale, and connection in more technical terms. He examines the relationships between urban components and offers a fundamentally different definition of a city, to clarify what type of city is fractal (Salingaros, 2004). In an article entitled "Comparing the morphology of urban patterns in Europe: a fractal approach" Frankhauser (2004) concludes that fractal model is a proper way to compare the diverse morphology of European cities. In the study, the model is used to identify the morphology of Brussels, Lyon, Stuttgart, Helsinki, Bergamo and Strasbourg (Frankhauser, 2004).

In the study, "Fractal characteristics of soils under different land-use patterns in the arid and semiarid regions of the Tibetan Plateau, China", using the fractal dimension of particle-size distributions, Wang et al. (2006) found a relationship between land use and fractal dimension of PSD in the said areas in China (Wang et al., 2006). Thomas et al. (2007), in their study entitled "Fractal dimension versus density of the built-up surfaces in the periphery of Brussels", compared fractal-based parameters and different methods of fractal calculations used in Brussels to explain spatial variations using geographical variables and urban economics and land use planning (Thomas et al., 2007). Jehnsen et al. (1994) reported the results of applying fractal methods and patterns in inner urban structures. In addition, in the article "fractal based design model for different architectural languages" Gozubuyuk et al. (2008) used box-counting method to analyze urban design and traditional buildings in the two historical regions of Istanbul and Mardin in Turkey in order to determine their Fractal dimensions.

"Thomas et al. (2008) in the article entitled "Is there a link between fractal dimension and residential environment at a regional level?", studied the statistical relationship between fractal dimension and residential satisfaction. The study showed theoretical relationships and statistical relationships between some fractal indices and built environment quality (Thomas et al., 2008). In the article "derivation of the functional relations between fractal dimension of and shape indices of urban form", Chen (2011) points to the relationship between size, scale, shape, and dimension of urban settlements as basic problems that should be solved in near future and hopes his study can help provide an achievable perspective for understanding those problems. Therefore, the relationships between the fractal dimension of urban boundary and the compaction ratio of the urban shape were investigated in the study using a simple geometric measure. Compression ratios were shown to be exponential functions of the reciprocal of the boundary dimension. The results could be generalized and applied to common shape indices including circularity ratio, ellipticity index, and form ratio as defined by urban area, perimeter, or Feret’s diameter. It can be concluded from the said study that certain functional relations exist between the shape indices on the one hand and the boundary dimension on the other. In addition, within certain range of scales, ratios of size measurements can be used to estimate fractal parameters indirectly in order to reflect the features of urban shapes (Chen, 2011). Batty (2011), in a study on the new science of cities, evaluated and compared similar dimensions of cities such as London, Tottenham and Birmingham based on satellite images. In his view, self-similarity is a decisive factor in urban morphology. He used satellite images taken in 1991-2000 and concluded that the old parts of the cities under study were more complex than the checkered and newly built sections.

Chen (2013) in his article "Fractal analytical approach of urban form based on spatial correlation function" states that though urban form has been empirically proven to be invariant in terms of scaling, can be described by fractal geometry. However, the rational range of fractal dimension value and the relationships among different fractal indicators in cities are not theoretically uncovered yet. He used mathematical transform and found that fractal concepts were associated with scaling analysis, spectral analysis, and spatial correlation analysis and can be incorporated into a novel approach to fractal analysis of cities. He called the method, ‘3S analyses’ of urban form, and used it to obtain a set of fractal parameter equations, to find the relationships among different fractal parameters of cities. Each fractal parameter has a reasonable range of values, and, according to the fractal parameter equations, the intersection of such ranges of different fractal parameters suggests the proper scale of the urban patterns’ fractal dimension, ranging from 1.5 to 2. Geographers benefit from 3S-analysis-based fractal dimension equations and the numerical relationships between different fractal parameters to understand urban evolution and future city planners can possibly use them in their future planning (Chen, 2013). Feng and Liu (2015) in their article "Fractal dimension as an indicator for quantifying the effects of changing spatial scales on landscape metrics" state that though geographers and ecologists are aware of the scale effects of landscape patterns, quantifying such effects is still required. They used the fractal method to appraise the scale (grain or cell size) of landscape metrics at landscape and class levels. To this end, they changed the polygon data of the original land use into raster data at eleven aggregate scales to evaluate the fractal dimensions. They witnessed a fractal law consistent over a range of scales for most landscape metrics in the area and edge, shape and the aggregation groups. Moreover, the scale effects were more complex at the class level compared to those at the landscape level. Using the fractal method for quantitative evaluation of the scale effects can be used as a basis for studying landscape patterns at the time of upscaling or downscaling and is also useful in understanding landscape patterns (Feng and Liu, 2015).

In their article, "Fractal behavior of traffic volume on urban expressway through adaptive fractal analysis", He et al. (2015) point to fractal analysis as a widely used tool in a variety of fields such as finance, physiology, precipitation, biological systems, language and culture, geophysics, etc. They aimed at applying adaptive fractal analysis to traffic volume. In their paper, the autocorrelation function adopted on traffic volume data showed a long-range correlation behavior in two situations investigated (He et al., 2015). In their study "fractal dimensions of urban border as a criterion for space management", Jevric and Romanovich (2016) state that today, the residents of large cities tend to be in the vicinity of open spaces for various reasons, e.g., reducing stress, noise pollution, traffic, etc. and such tendency even exists in Podgorica (Montenegro). This claim was confirmed by the results of questionnaires completed in their study. Information on the fractal dimensions of urban boundaries and urban areas can be used as a parameter for making decisions related to spatial development. In this way, suburban areas that form urban boundaries can be compacted but this should not be done at the expense of their contact with open or green space (Jevric and Romanovich, 2016). In a study entitled "the role of city size and urban form in the surface urban heat island", Zhou et al. (2017) investigated the effect of urban form and size on heat islands and found that the intensity of heat islands among 5,000 large cities was directly associated with city size and urban fractal dimensions, but was inversely related to the anisometry logarithm of the cities. City size had a significant effect on heat islands; therefore, small towns had fewer heat island cores than did stretched and scattered cities (Zhou et al., 2017).

In the study, "spatial analysis of cities using Renyi entropy and fractal parameters", Chen and Feng (2017) divide spatial distribution of cities into two groups: 1) simple distribution with characteristic scale such as exponential distribution; and 2) complex distributions without uncharacteristic scale like power-law distribution. The second case belongs to scale-free distributions that can be modeled using fractal geometry. However, the fractal dimension is not suitable for the first type of distribution. In contrast, spatial entropy can be used to measure any type of urban distribution. Thy aimed at using the dual relation between Euclidean and fractal geometries to generalize multi-fractal parameters. They used mathematical derivation and empirical analysis as their main methods, and used the fact that the normalized fractal dimension is equal to the normalized entropy, as their theoretical foundation. On this basis, they defined spatial indexes called “dummy multifractal parameters” for geographical analysis. Such indexes could be used to describe both simple and complex distributions. They applied dummy multifractal indexes to investigate the population density distribution of Hangzhou city, China. The results showed the spatio-temporal evolution of urban morphology in Hangzhou. The study indicated the usefulness of combining fractal dimension and spatial entropy to achieve a new method for spatial analysis of city development. An objective of the study was to link the traditional 'scale-based spatial analysis and future scaling-based spatial analyses of cities to create a methodological framework for urban studies (Chen and Feng, 2017). In a study entitled "Temperature-Humidity Index described by fractal Higuchi Dimension affects tourism activity in the urban environment", Ana Maria et al (2018) used fractal index to investigate the effects of these indicators on tourism. The study area was Focşani City in Romania studied from 2001 to 2016. The results of studies showed that high temperature and humidity directly affected tourism and Higuchi Dimension (DH) could best show the monthly and annual effects of temperature and humidity on tourism. The study also identified indicators with direct negative effects on tourism (Ana-Maria et al., 2018).

In the study, "spatial-temporal fractal of urban agglomeration travel demand", He (2020) specified spatial-temporal networks for travel demand through spatial-temporal analysis and explicit analysis of network characteristics. The results showed that the degree distributions of the networks of spatial–temporal travel demand follow power laws, which decreased with the growth of the square cells used to 'divide an urban agglomeration to aggregate travel demand'. However, following power laws was not affected by 'the spatial–temporal granularity of network construction'. This showed that urban agglomeration travel demand had a spatial–temporal fractal nature. These findings can help us understand the nature of travel demand as well as human movement across an urban agglomeration (He, 2020). Gong et al. (2020) carried out a study entitled "Research on the complexity of form and structure of green spaces based on fractal models" state that the fractal nature of urban green spaces is the product of the self-organizing evolution of complex urban systems into higher levels and patterns. The regular and complex structures of urban green space emerge after a certain stage of development. Based on GF1 satellite data collected in 2019 and three fractal models, the complexity of the shapes and structures of a green space system in downtown Dalian, China were examined. The results showed that the boundary dimension measured by the perimeter-scale model was 0.64-1.40 and the boundary dimension measured by the area-perimeter model was 1.79-1.99. These indicated a high degree of human disturbance in the boundaries of green space, and a poor stability of green space spatial structures (Gong et al., 2020).

Among studies carried out in Iran, Mohajeri (2007), in a study entitled "fractal city, the language of nature in urban design", studied the city of Yazd based on the laws governing fractals and showed the city's compliance with fractal laws. In the dissertation entitled "Fractal geometry, complexity and the nature of urban morphological evolution: developing a fractal anaylsis tool to assess urban morphological change at neighborhood level", Haghani (2009) showed the failure of Euclidean geometry in understanding the city geometry and the need for applying the theory of complexity and fractal geometry in measuring city morphology, and using the fractal dimension as a decisive characteristic to depict the complexity of urban contexts. He examined the context of District 1 in Tehran and specifically analyzed the experimental changes over time using fractal dimension. Based on fractal dimensions obtained via Benoit 1.3 for the neighborhood units in the region, he demonstrated the reduction of complexity over time in the context and concluded that the fractal dimension, as a unique characteristic, should be considered as a determining factor in zoning urban contexts. Madani Isfahani (2011), in a study entitled "Urban space design, an intervention in a complex system with emphasis on geometric fractal dimensions in the urban landscape", investigated three samples of Isfahan Bazaar, Chaharbagh Abbasi Street as the main structure of Isfahan, and Olfat Street is the main axis of the new city of Baharestan in terms of fractal geometry. Then, according to characteristics of organic structure of markets, the actual and potential characteristics in its perspective were analyzed and compared with two other samples. Finally, using the results obtained from the geometric analysis of market bodies, visual frameworks and patterns were provided for new designs or arrangements.

In a study entitled "a comparative study of morphology of old and new urban contexts based on fractal model: the case of selected neighborhoods in old and new contexts of Zanjan" Heidari (2016) introduced the fractal model and its role in the study of the old context and the new city of Zanjan. Findings showed that the fractal model was a real and accurate tool for understanding the form of cities and that the old context of Zanjan is more fractal compared to the new one. In another study entitled "Study of urban context dispersion using fractal geometry model and complexity theory to find urban development patterns in Tehran's 20th district", Goodarzi et al. (2015) attempted to find the urban sprawl pattern taking into account the indicators of density, area of built environments, etc. The study investigated horizontal development and urban growth, and then measured fractal dimension using GIS techniques along with statistical indicators and aerial photographs. The results showed that among the areas of the southern part of Tehran, District 20 sprawled towards the surrounding villages in the south getting close to Shahriar and Shahre Quds to the west. The study of the degree of sprawl and complexity of the district using satellite images of 2006 and 2011 and fractal dimension calculations showed a direct relationship between the amount of complexity and urban sprawl. Mirkatouli et al. (2014), in their study entitled "explaining different aspects of using fractal geometry in geographical analysis and urban planning", introduced fractal geometry and the effects of fractal dimension on the city as a whole and its elements. In addition, the relationship between fractals in geography and its sub-disciplines (physical and human geography) was discussed and the role and application of fractals in geography and urban planning were presented. In the meantime, the characteristics and typology of cities with fractal structure built and formed throughout history and the effects of fractals on the development of cities were pointed out, and finally the implications for planning and designing urban spaces based on geometry of fractals with a focus on geography. In a dissertation entitled "the effects of external constraints on the overall shape and internal geometric patterns of cities", Mohajeri (2012) examined city growth over time by focusing on the network of roads and the direction of their expansion, and showed the geographical and natural factors limiting and controlling the city growth using Georient software. The results proved that the size and direction of the road network and consequently the shape of the city were affected by geographical factors. Then, using power-scale law and dividing the road network into specific categories based on cumulative frequency, the compliance of the road network with the power-scale law was shown. According to the obtained results, there was a significant relationship between the length of the road network and the typology of the network of passages based on their functions (local, collector and distributor, main arterial roads, etc.) in a scientific way. Based on what was mentioned in this section, the results and findings of studies related to urban fractals will be discussed below.

2.4. Classification of Previous Studies

In this section, in order to answer the questions raised at the beginning of the systematic review, a comprehensive classification of various dimensions was done based on the thorough search presented in the previous section. In response to the first question, i.e. "What are the developing and emerging concepts in theoretical and experimental studies that can explains the role and applications of fractal theory in the study and simulation of the built environment?", different studies were classified in 10 categories including 1) psychology, 2) architecture, 3) urban design, 4) urban form and structure, 5) urban boundary shape, 6) land use, 7) urban traffic, 8) modeling urban growth, 9) urban hierarchy and 10) fractal benefits. Each category includes more detailed research sub-fields (Table 1).

Table 1 Classification of articles by different fields of study (Source: Authors)

Field of study	Research sub-fields	Researchers	Number	Share of the total
Psychology	Psychological effects of fractal forms on individuals	Abboushi et al., 2019 Taylor, 2006 Chalup, et al., 2008 Pihel, 2011 Lorenzos et al., 2017 El-Darwish, 2019	6	4.7
	Investigating the relationship between fractal dimension and residential environment satisfaction	Thomas et al., 2008 Cavailhès et al., 2009	2	1.6
	Tourism activities in urban areas	Ana-Maria et al., 2018	1	0.8
Architecture	Identifying the applications of fractal forms in architecture (layout and order, hidden structural advantages)	Sakai et al., 2012 Burkle-Elizondo, 2001 Burkle-Elizondo and Valdéz-Cepeda, 2006 Gozubuyuk et al., 2008 Capo, 2004 Rian and Asayama, 2016	4.7	6
Urban Design	Identifying applications of fractal forms in urban design	Lu et al., 2012a, 2012b Robertson, 1995 Batty and Longley, 1994 Bovill, 1996 Cooper, 2005 Mohajeri, 2007 Madani Isfahani, 2011	8	7.9
	Relationships among urban components	Salingaros, 2004	1	0.8
	Investigating the concept of compactness ratio and fractal dimension	Wang et al., 2006	1	0.8
	Urban Morphology	Frankhauser, 2004 Batty, 2011 Tannier and Thomas, 2013 Weber, 2001	4	3.2
	Investigation of the relationship between fractal and urban density	Thomas et al., 2007 Batty, 1995 Frankhauser, 1998	3	2.3
Urban form and structure	Studies of urban and regional form and structure and urban distribution	Batty and Longley, 1994 Chen, 2011, 2013, 2017 Jevric and Romanovich, 2016 Batty et al., 1989 Jehnsen et al., 1994 Cort, 2013 Alam, 2018 Sogo et al., 2018 Gong et al., 2020 Heidari, 2016 Goodarzi et al., 2015 Mohajeri, 2012	14	11.1

	Urban scale studies	Chen, 2013 Jehnsen et al., 1994 Zhou et al., 2017	3	2.4
	Urban rank-size distribution	Chen and Zhou, 2004 Dendrinos and El Naschir, 1994	2	1.6
	Investigating the concept of entropy ratio and fractal dimension	Chen, 2012, 2017, 2020 Chen et al., 1991, 1993 Feder, 1988 Ryabko, 1986 Feng and Chen, 2010 Batty, 2010 Batty et al., 2014 Hong et al., 2017 Kornejady and Pourghasemi, 2019	12	9.5
	Qualities of symmetry, self-similarity and diversity and difference in scale	Kiani and Amiriparyan, 2016 Sala, 2003 Chen, 2011	3	2.4
	Investigating the concept of complexity in fractal forms	Rian and Asayama, 2016 Batty et al., 1989 Liu and Chen, 2007 Chen, 2017 Kacha et al, 2013 White and Engelen, 1993, 1994 Batty, 2005 Gong et al., 2020 Jevric and Romanovich, 2016 Kornejady and Pourghasemi, 2019 Sezer, 2010	12	9.5
Urban boundary shape	Boundary dimension and urban border shape	Tannier and Thomas, 2013 Jevric and Romanovich, 2016 Gong et al., 2020 Batty and Longley, 1987, 1994 Longley and Batty, 1989 Frankhauser, 1994 Longley et al., 1991 Chang, 1996 Chang and Wu, 1998 Chen and Wang, 2016 Chen, 2013 Olsen et al., 1993	13	10.3
Land use	Urban land use studies	Batty and Longley, 1994 Gong et al., 2020 Barr and Barnsley, 1997	3	2.4
Urban Traffic	Fractal behavior of urban traffic volume	He et al., 2015	1	0.8
	Intra-city travel demand	He, 2020	1	0.8
Urban Growth	Remote sensing and urban growth models for understanding urban	Herold et al., 2001, 2003 Chen, 2013	5	3.9

Modeling	development	Jiang and Jia, 2011 Rozenfeld et al., 2008		
	Spatial development decision making	Jevric and Romanovich, 2016	1	0.8
Urban hierarchy	Urban and regional hierarchy	Batty et al., 1989 Matsuba, 2003 Chen and Zhou, 2004 Tannier and Thomas, 2013	4	3.2
	Searching for fractal geometry at different levels	Kiani and Amiriparyan, 2016 Chen and Zhou, 2004 Feng and Liu, 2015 Kaye, 1989 Wong and Fotheringham, 1990 Arlinghaus, 1985	6	4.8
Benefits of fractal	Identifying the properties and benefits of fractal forms	Rian and Asayama, 2016 Batty and Longley, 1994 Mandelbrot, 1983 Frankhauser, 1994 Chen, 2005 Chen and Huang, 2019 Anas et al., 1998 Lagarias, 2007 Cooper and Oskroch, 2008 Crompton, 2001 Kim et al., 2003 Mirkatouli et al., 2014	12	9.5

According on what has been said about the research fields extracted from the studies conducted on fractals, the percentage of the share of each of the 10 fields according were determined (Figure 1).

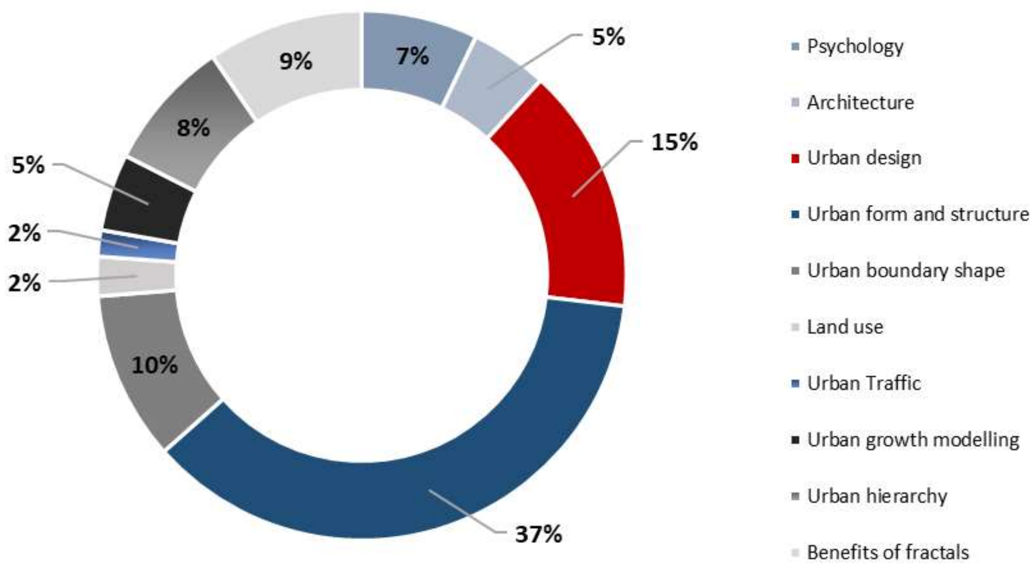

Figure 1: The share of each of the 10 fractal research fields extracted from the systematic review, Source: Authors

As shown in the diagram, studies related to the field of urban form and structure had the highest share of studies with 37%, followed by urban design (15%) and urban boundary shape (10%) and benefits of fractals (9%) as the most studied areas, respectively. In contrast, urban traffic (2%) and land use (2%) have been less studied compared to other areas. In the following, the share of each research sub-field is shown graphically (Charts 2 to 7).

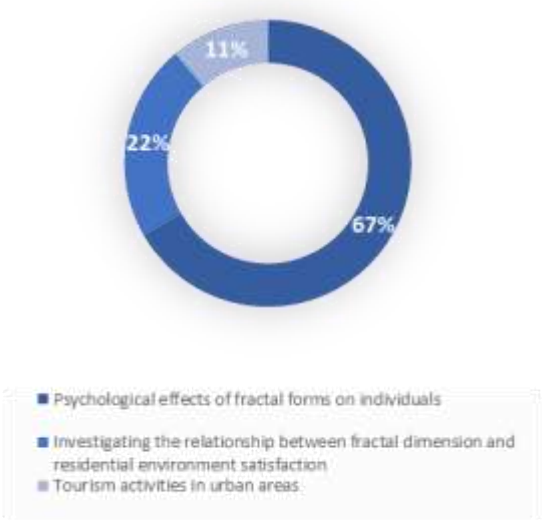

Fig 2 Share of research sub-fields in psychology

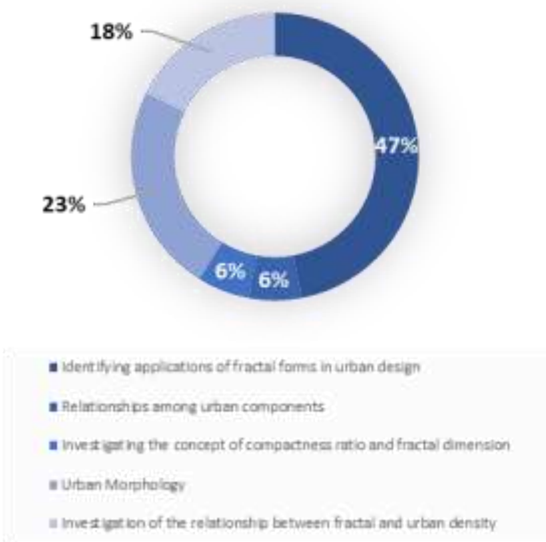

Fig 3 Share of research sub-fields in urban design

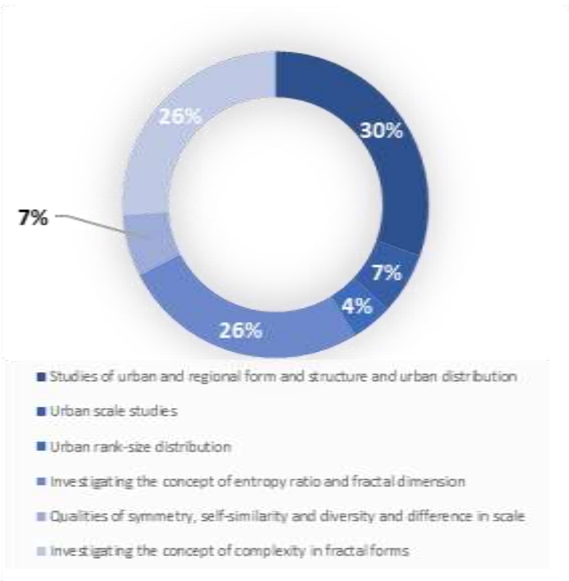

Fig 4 Share of research sub-fields in urban form and structure

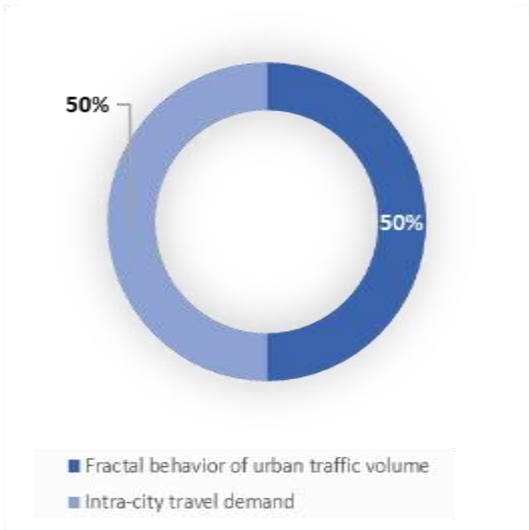

Fig 5 Share of research sub-fields in urban traffic

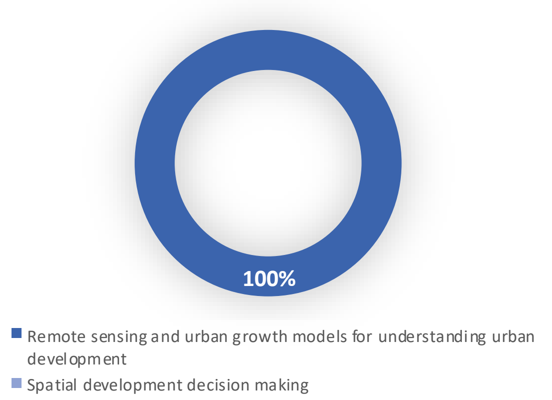

Fig 6 Share of research sub-fields in urban growth modeling

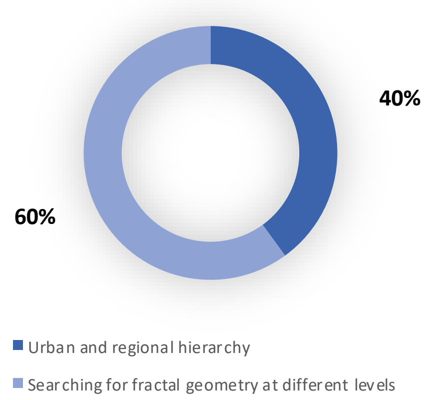

Fig 7 Share of research sub-fields in urban hierarchy

In response to the second question posed in the systematic review, i.e. "What are the key theories and methods that have led to the development of fractal studies in the architecture, design and urban planning literature?", the review of methods, measures and dimensions used in the studies showed that fractal dimension and Hausdorff dimension (34%), boundary dimension (24%), network or box counting dimension (14%) and power-law distribution, power-law and power-scale laws (7%) had larger shares (Table 2).

Table 2 Classification of methods, measures and dimensions used in the systematic review (Source: Authors)

Methods, measures and dimensions	Researchers	Number	Share out of the total
Circularity coefficient, ellipticity index and form ratio defined by urban area, perimeter or Feret's diameter	Chen, 2011 Kacha et al, 2013	2	3.6
Fractal and Hausdorff dimensions	Thomas et al., 2008 Bovill, 1994 Wang et al., 2006 Thomas et al., 2007 Tannier and Thomas, 2013 Chen, 2013 Feng and Liu, 2015 Jevric and Romanovich, 2016 Zhou et al., 2017 Chen and Huang, 2019 Anas et al., 1998 Lagarias, 2007 Balankin, 2020 Jimenez et al., 2019 Abboushi et al., 2019 Haghani, 2009 Madani Isfahani, 2011 Goodarzi et al., 2015 Mirkatouli et al., 2014	19	34.5

Standard circle and radius dimension	Chen, 2011 Frankhauser and Sadler, 1991 Chen et al., 1991	3	5.5
Self-similarity	Sala, 2003	1	1.8
Three-parameter Zipf model	Chen and Zhou, 2004	1	1.8
Grid dimension (box dimension), box counting method or raster analysis	Gozubuyuk et al., 2008 Benguigui et al., 2000 Chen et al., 1991 Chen, 2013 Gong et al., 2020 Frankhauser, 1998 Kacha et al, 2013 Sezer, 2010	8	14.5
3S Analysis	Chen 2013	1	1.8
Sprawl and complexity	Goodarzi et al., 2015	1	1.8
Adaptive Fractal	He et al., 2015	1	1.8
Power law (PL) distributions, power laws, and power-scale law	Chen, 2017 Chen et al., 1993 He, 2020 Mohajeri, 2012	4	7.3
Spatial Entropy	Chen, 2017	1	1.8
Boundary dimension	Gong et al., 2020 Tannier and Thomas, 2013 Jevric and Romanovich, 2016 Batty and Longley, 1987 Longley and Batty, 1989 Batty and Longley, 1994 Frankhauser, 1994 Longley et al., 1991 Chang, 1996 Chang and Wu, 1998 Chen and Wang, 2016 Chen, 2013 Olsen et al., 1993	13	23.6

The share of each of the methods, measures, and dimensions used in the studies examined is shown in the Figure 8.

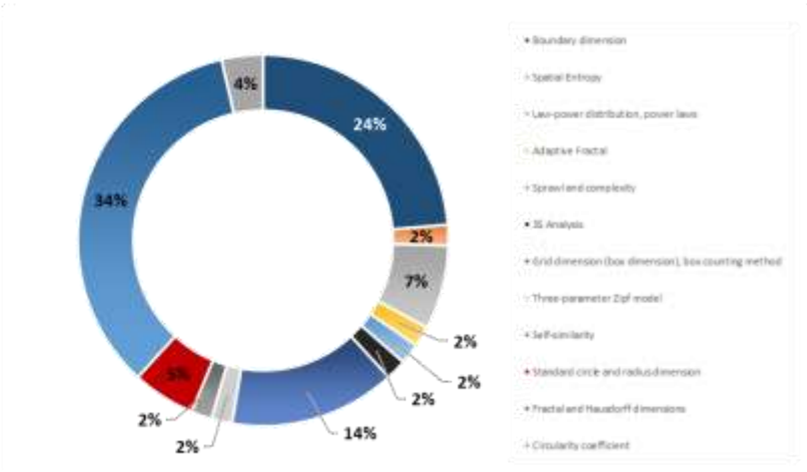

Fig 8 Share of each of the methods, measures, and dimensions used in studies included in the systematic review (Source: Authors)

3. Conclusion

As stated, new knowledge about complex systems has given rise to a new perspective to cities that might no longer be seen as a simple, organized system with linear geometry and structure, but as a complex organism demonstrating a special regularity through its shape at different scales. Fractal geometry is a powerful tool in geographical modeling and spatial analysis and has long been used in urban studies. Research shows that "cities are ideal candidates for fractal analysis", indicating that cities are formed through the gradual influence of local factors that create organized urban patterns.

Urban systems literature involves pioneering studies on the development and use of fractal geometry to understand and plan the physical shape and spatial network of cities. Fractal theory enables the mathematical modelling and computer simulation of urban systems. Different studies have shown how cities grow in ways that seem irregular at first glance, but show an underlying order demonstrating their complexity and diversity, when examined in terms of fractals. Research on fractal cities has witnessed a significant progress in various aspects, including urban forms, structures, transportation, and the dynamics of urban growth. Therefore, given the deep and wide-ranging applications of fractal theory in explaining local and spatial complexities, this study raised the question of what practical and theoretical frameworks useful for specialists in the fields of geography and urban planning can be achieved via a systematic review and content analysis of research conducted on fractals? With the aim of summarizing, interpreting, and understanding the patterns of findings in previous research, this study systematically reviewed and performed content analysis of existing studies in the selected field of research.

To this end, in the first stage, i.e., collection, the research problem or question was defined and criteria for study selection and systematic search of related resources were specified. The second stage, i.e., evaluation, involved reviewing the validity of studies, summarizing findings, and extracting data systematically. In the third stage; i.e., synthesis, descriptive analysis of studies, analysis, and interpretation of studies via descriptive methods and graphic tools were carried out. In addition, during the study, two research questions were posed. The first question was: What are the developing and emerging concepts in theoretical and experimental studies that can explains the role and applications of fractal theory in the study and simulation of the built environment? In order to answer this question, different research fields were classified in 10 categories including 11) psychology, 2) architecture, 3) urban design, 4) urban form and structure, 5) urban boundary shape, 6) land use, 7) urban traffic, 8) modeling urban growth, 9) urban hierarchy and 10) benefits of fractal, each containing more detailed research subfields. Among them, studies related to urban form and structure had the highest share of studies with 37%, followed by urban design (15%) and urban boundary shape (10%) and benefits of fractals (9%) as the most studied areas, respectively.

The second question was: "What are the key theories and methods that have led to the development of fractal studies in the architecture, design, and urban planning literature? In response, based on the methods, measures, and dimensions used in the reviewed studies, it can be concluded that fractal dimension and Hausdorff dimension, boundary dimension, grid dimension or box counting dimension, and power law (PL) distributions, power laws, and power-scale law had larger shares of application in the studies, respectively.

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